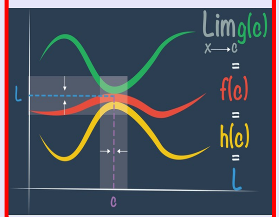
Math 261

Spring 2023

Lecture 34



Feb 19-8:47 AM

If $f(x)$ is differentiable at $x=a$, then it is continuous at $x=a$.

Since $f(x)$ is differentiable at a

$\Rightarrow f'(a)$ exists.

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

Recall

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

If $x=a$

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

and $h\to 0$ $x-a=h$

$$f(x)-f(a)=\frac{f(x)-f(a)}{x-a}\cdot(x-a)$$

$$\lim_{x\to a}\left[f(x)-f(a)\right]=\lim_{x\to a}\left[\frac{f(x)-f(a)}{x-a}\cdot(x-a)\right]$$

$$=\boxed{\lim_{x\to a}\frac{f(x)-f(a)}{x-a}}\cdot\lim_{x\to a}(x-a)$$

$$=f'(a)\cdot\lim_{x\to a}(x-a)=f'(a)\cdot(a-a)$$

$$=f'(a)\cdot 0$$

$$=\boxed{0}$$

$$\lim_{x\to a}\left[f(x)-f(a)\right]=0$$

$$\lim_{x\to a}f(x)-\cancel{\lim_{x\to a}}f(a)=0$$

$f(a)$

$$\lim_{x\to a}f(x)-f(a)=0$$

$$\boxed{\lim_{x\to a}f(x)=f(a)}$$

$f(x)$ is continuous at $x=a$.

Apr 17-8:47 AM

Rolle's Theorem:

Suppose

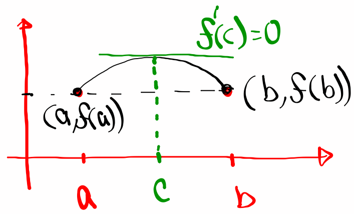
1) $f(x)$ is continuous on $[a,b]$,

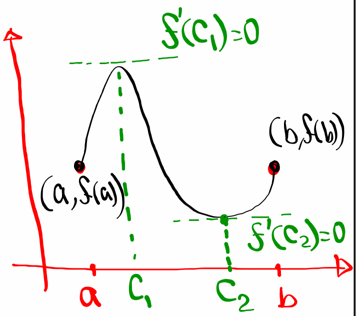
2) $f(x)$ is differentiable on (a,b),

3) $f(a)=f(b)$

then there is a number c in (a,b)

Such that $f'(c)=0$





Apr 17-8:59 AM

Mean-Value Theorem:

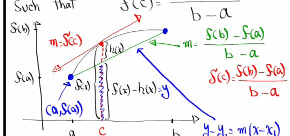
Suppose

1) $f(x)$ is continuous on $[a,b]$,

2) $f(x)$ is differentiable on (a,b),

then there is a number c in (a,b)

Such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$



$$y-f(a)=\frac{f(b)-f(a)}{b-a}(x-a)$$

$$y=f(a)+\frac{f(b)-f(a)}{b-a}(x-a)$$

$$f(x)-h(x)=f(a)+\frac{f(b)-f(a)}{b-a}(x-a)$$

If we isolate $h(x)$,

$$h(x)=f(x)-f(a)-\frac{f(b)-f(a)}{b-a}(x-a)$$

$$h(a)=\cancel{f(a)}-\cancel{f(a)}-\frac{f(b)-f(a)}{b-a}\cdot\cancel{(a-a)}^{\,0}=0$$

$$h(b)=f(b)-f(a)-\frac{f(b)-f(a)}{\cancel{b-a}}\cdot\cancel{(b-a)}$$

$$=\cancel{f(b)}-\cancel{f(a)}-\cancel{f(b)}+\cancel{f(a)}=0$$

$h(a)=h(b)$ $\qquad h(x)=f(x)-f(a)-\dfrac{f(b)-f(a)}{b-a}\cdot(x-a)$

$$h'(x)=f'(x)-0-\frac{f(b)-f(a)}{b-a}\cdot 1$$

$$h'(x)=f'(x)-\frac{f(b)-f(a)}{b-a}$$

Since $h(a)=h(b)$ and $h(x)$ is cont. on $[a,b]$, and $h(x)$ is diff. on (a,b), by Rolle's theorem

$h'(c)=0$, $\quad f'(x)-\dfrac{f(b)-f(a)}{b-a}=0$ at $x=c$

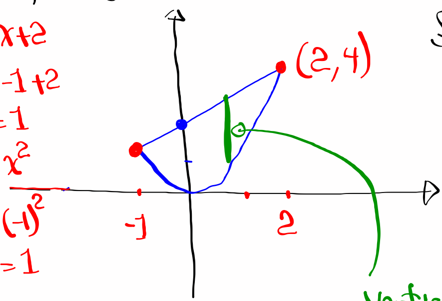
$$f'(c)-\frac{f(b)-f(a)}{b-a}=0$$

MVT $\qquad f'(c)=\dfrac{f(b)-f(a)}{b-a}$

Apr 17-9:07 AM

Graph $y = x+2$ & $y = x^2$ for $-1 \le x \le 2$.

$y = x+2$
$= -1+2$
$= 1$
$y = x^2$
$= (-1)^2$
$= 1$



Find the maximum vertical distance between these two graphs.

Vertical distance
$y_{line} - y_{curve}$

max $x+2 \quad - x^2$

let $f(x) = x+2-x^2$

$f'(x) = 1 - 2x$

$f''(x) = -2$

$f'(x) = 0$

$1-2x = 0 \quad x = \frac{1}{2}$

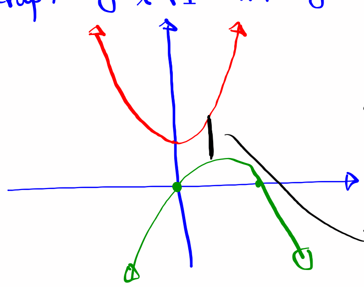
what is the vertical distance?

$\frac{1}{2} + 2 - \left(\frac{1}{2}\right)^2 = \frac{1}{2} + 2 - \frac{1}{4} = \frac{1}{4} + 2$

$= \boxed{2.25}$

Apr 17-9:29 AM

Graph $y = x^2+1$ and $y = x - x^2$. $y = x - x^2$
$= x(1-x)$



Find minimum vertical distance between them.

$y_{Red} - y_{Green}$

$x^2+1 - (x - x^2)$
$= x^2 + 1 - x + x^2$
$= 2x^2 - x + 1$

$f(x) = 2x^2 - x + 1$

$f'(x) = 4x - 1$

$f''(x) = 4$ C.U.

min.

$f'(x) = 0$
$4x - 1 = 0$
$x = \frac{1}{4}$

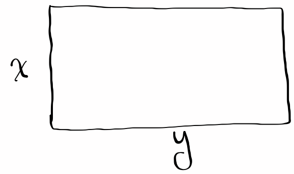
Min. distance
$f\left(\frac{1}{4}\right) = 2\left(\frac{1}{4}\right)^2 - \frac{1}{4} + 1 = 2 \cdot \frac{1}{16} - \frac{1}{4} + 1$

$= \frac{1}{8} - \frac{1}{4} + 1$

$= \frac{-1}{8} + 1 = \boxed{\frac{7}{8}}$

Apr 17-9:37 AM

Find dimensions of the rectangle with largest area with perimeter of 100 m.



$P = 100$

$2x + 2y = 100$

$x + y = 50 \rightarrow y = 50 - x$

Area xy

$x(50 - x)$

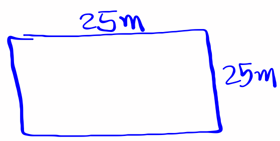
$A(x) = x(50 - x)$

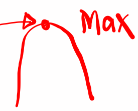
$= 50x - x^2$

$A'(x) = 50 - 2x$

$A''(x) = -2 < 0$ C.D.

Area $= xy = 25(25) = \boxed{625\ m^2}$





$A'(x) = 0$

$\boxed{x = 25}$

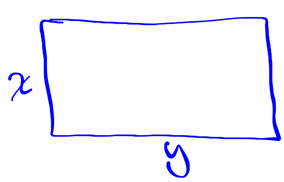
$y = 50 - 25 = \boxed{25}$

Apr 17-9:45 AM

Find dimensions of a rectangle with area 1000 m^2 whose $\boxed{\text{perimeter}}$ is as Small as Possible.

$A = 1000$

$xy = 1000 \rightarrow y = \frac{1000}{x}$



$2x + 2y$ Minimum

$2x + 2\left(\frac{1000}{x}\right)$

$f(x) = 2x + \frac{2000}{x}$

$f'(x) =$

$f''(x) =$

Apr 17-9:52 AM